TEMPERATURE FIELD IN A SPHERICAL PARTICLE

DURING QUASISTEADY HIGH-INTENSITY DRYING

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Equations for calculating the temperature field in a spherical particle during convective drying in a gaseous medium with variable parameters are derived.

Recently, dryers operating in forced hydrodynamic modes according to principle of stream whirling are used more widely in the chemical industry mainly for the drying of materials with surface moisture. The short period of time such a material remains in the dryer chamber and the relatively high velocities of the interacting streams make it feasible to appreciably intensify the drying process and to raise the temperature of the drying agent. These advantages have been fully realized in the compound dryers [1-3] developed at the Chair of Processes and Apparatus at the Ivanovo Institute of Chemical Technology and have already been operating successfully in the industry for several years.

An important step in the determination of the optimum performance parameters and in the design of the apparatus is calculation of the temperature field in a particle moving through a gaseous medium with a variable temperature. The solution to analogous problems in the case of pure heat transfer is well known [4, 5]. In this study the problem of heat transfer involving a spherical particle in a gas stream with a variable temperature, as well as mass transfer, will be solved.

In the calculation of drying processes one usually assumes that during evaporation of the surface moisture the temperature of a particle is equal to the temperature of adiabatic air saturation. This assumption is correct in most cases. In the case of high-intensity drying, however, the balance of heat supplied to a particle and removed from it together with the stream of moisture can become disturbed. When the amount of heat supplied exceeds the amount removed with evaporated moisture, then a temperature gradient appears in the particle: the surface temperature rises and this causes the motive force of the evaporation process to increase. If we regard the drying of a disperse material in a compound apparatus as a quasisteady process, we have, for the temperature field in a single particle in a medium with a variable temperature, the boundary-value problem

$$\frac{\partial t(r,\tau)}{\partial\tau}=a\left[\frac{\partial^2 t(r,\tau)}{\partial r^2}+\frac{2}{r}\frac{\partial t(r,\tau)}{\partial r}\right],\ \tau>0,\ 0\leqslant r\leqslant R, \tag{1}$$

$$t(r,0)=t_0, \tag{2}$$

$$\frac{\partial t(0,\tau)}{\partial r}=0, \tag{3}$$

$$\alpha[t_c(\tau)-t(R,\tau)=\lambda\frac{\partial t(R,\tau)}{\partial r}+\beta\Delta Pr^*. \tag{4}$$

We change variables in Eqs. (1)-(4), using the notation

$$T(r,\tau)=t(r,\tau)-t_0, \tag{5}$$

$$T_a(\tau)=t_a(\tau)-m\Delta Pr^*-t_0. \tag{6}$$

Then the boundary-value problem becomes

$$\frac{\partial T(r,\tau)}{\partial\tau}=a\left[\frac{\partial^2 T(r,\tau)}{\partial r^2}+\frac{2}{r}\frac{\partial T(r,\tau)}{\partial r}\right],\ \tau>0,\ 0\leqslant r\leqslant R, \tag{7}$$

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$$T(r, 0) = 0, \tag{8}$$

$$\frac{\partial T(0, \tau)}{\partial r} = 0, \tag{9}$$

$$\alpha [T_a(\tau) - T(R, \tau)] = \lambda \frac{\partial T(R, \tau)}{\partial r}. \tag{10}$$

With the aid of Duhamel's theorem [4, 6], we write the solution to the system of equations (7)-(10) as

$$T(r, \tau) = T_a(\tau)\psi(r, 0) + \int_0^\tau T_a(\theta) \frac{\partial \psi(r, \tau - \theta)}{\partial \tau} d\theta, \tag{11}$$

where $\psi(r, \tau)$ is the solution to the problem for $T_a = 1$.

Applying the Laplace integral transformation [6] to Eqs. (7)-(10) yields for the function $\psi(r, \tau)$

$$\tilde{\psi}(r, s) = \frac{N_{Bi} R \sinh \sqrt{s/a}\, r}{rs[(N_{Bi} - 1)\sinh \sqrt{s/a} R + \sqrt{s/a}\, R \cosh \sqrt{s/a} R]}, \tag{12}$$

$$\psi(r, \tau) = 1 - \sum_{n=1}^{\infty} A_n \frac{R \sin \mu_n r/R}{r} \exp\left(-\mu_n^2 \frac{a\tau}{R^2}\right), \tag{13}$$

where

$$A_n = \frac{2(\sin \mu_n - \mu_n \cos \mu_n)}{\mu_n(\mu_n - \sin \mu_n \cos \mu_n)} \tag{14}$$

and μ_n are roots of the characteristic equation $\tan \mu = \mu/(1 - N_{Bi})$.

Relation (13) reduces expression (11) to

$$T(r, \tau) = T_a(\tau) - \sum_{n=1}^{\infty} A_n \frac{R \sin \mu_n r/R}{r} \left\{T_a(\tau) - \mu_n^2 \frac{a}{R^2} \int_0^\tau T_a(\theta) \exp\left[-\mu_n^2 \frac{a(\tau - \theta)}{R^2}\right] d\theta\right\}. \tag{15}$$

In dimensionless form expression (15) becomes

$$\bar{T}(\bar{r}, N_{Fo}) = 1 - \sum_{n=1}^{\infty} A_n \frac{\sin \mu_n \bar{r}}{\bar{r}} \left[1 - \mu_n^2 \int_0^{N_{Fo}} T_a(N_{Fo}^*) \exp(-\mu_n^2 N_{Fo,m})\, dN_{Fo}^*\right]. \tag{16}$$

Expressions (15) and (16) yield the temperature field in a particle dried by a stream of a heat carrier whose temperature is a function of time.

It has been said earlier that drying in a whirled high-velocity stream occurs at a high intensity, with the material remaining in the dryer chamber for a short period of time only. Meanwhile, it is also well known [6] that the convergence of the infinite series in expressions (15) and (16) becomes worse as τ decreases. This creates difficulties in the calculation, inasmuch as a large number of terms in the series must be retained. Here we will obtain a solution convenient for small values of the Fourier number.

Considering that $N_{Fo} \to 0$, while $\sinh x \approx \cosh x \approx 0.5e^x$, we transform expression (12) to

$$\tilde{\psi}(r, s) = \frac{N_{Bi} R}{rs\left[(N_{Bi} - 1) + \sqrt{\frac{s}{a}} R\right]} \left\{\exp\left[-\sqrt{\frac{s}{a}}(R - r)\right] - \exp\left[-\sqrt{\frac{s}{a}}(R + r)\right]\right\}. \tag{17}$$

A changeover to the domain of originals with the aid of tables of integral transforms [6] yields

$$\psi(\bar{r}, N_{Fo}) = \frac{N_{Bi}}{\bar{r}(N_{Bi} - 1)} \left\{\operatorname{erfc} \frac{1 - \bar{r}}{2\sqrt{N_{Fo}}} - \exp[(N_{Bi} - 1)^2 N_{Fo} + (N_{Bi} - 1)(1 - \bar{r})] \times\right.$$

$$\times \operatorname{erfc}\left[\frac{1-\bar{r}}{2\sqrt{N_{Fo}}}+(N_{Bi}-1)\sqrt{N_{Fo}}\right]-\operatorname{erfc}\frac{1+\bar{r}}{2\sqrt{N_{Fo}}}+$$

$$+\exp[(N_{Bi}-1)^2 N_{Fo}+(N_{Bi}-1)(1+\bar{r})]\operatorname{erfc}\left[\frac{1+\bar{r}}{2\sqrt{N_{Fo}}}+(N_{Bi}-1)\sqrt{N_{Fo}}\right]\Bigg\}. \quad (18)$$

Using again Duhamel's theorem, we obtain an expression for the temperature field in a particle during a short drying period

$$\bar{T}(\bar{r}, N_{Fo})|_{N_{Fo}\to 0}=\frac{N_{Bi}}{\bar{r}(N_{Bi}-1)\sqrt{\pi}}\int_0^{N_{Fo}} T_a(N_{Fo}^*)\frac{\psi_1(\bar{r}, N_{Fo,m})}{N_{Fo,m}}\,dN_{Fo}^*, \quad (19)$$

with function $\psi_1(\bar{r}, N_{Fo,m})$ calculated as

$$\psi_1(\bar{r}, N_{Fo,m})=\frac{1}{2\sqrt{N_{Fo,m}}}\Bigg\{(1-\bar{r})\exp\left[-\frac{(1-\bar{r})^2}{4N_{Fo,m}}\right]-(1+\bar{r})\times$$

$$\times\exp\left[-\frac{(1+\bar{r})^2}{4N_{Fo,m}}\right]\Bigg\}+\sqrt{\pi}\,(N_{Bi}-1)^2 N_{Fo,m}\Big\{\exp[(N_{Bi}-1)^2 N_{Fo,m}+$$

$$+(1+\bar{r})(N_{Bi}-1)]\operatorname{erfc}\left[\frac{1+\bar{r}}{2\sqrt{N_{Fo,m}}}+(N_{Bi}-1)\sqrt{N_{Fo,m}}\right]-\exp[(N_{Bi}-1)^2 N_{Fo,m}+$$

$$+(1-\bar{r})(N_{Bi}-1)]\operatorname{erfc}\left[\frac{1-\bar{r}}{2\sqrt{N_{Fo,m}}}+(N_{Bi}-1)\sqrt{N_{Fo,m}}\right]\Big\}-\Big[(N_{Bi}-1)\sqrt{N_{Fo,m}}-$$

$$-\frac{1+\bar{r}}{2\sqrt{N_{Fo,m}}}\Big]\exp\Big\{(N_{Bi}-1)^2 N_{Fo,m}+(1+\bar{r})(N_{Bi}-1)-$$

$$-\left[\frac{1+\bar{r}}{2\sqrt{N_{Fo,m}}}+(N_{Bi}-1)\sqrt{N_{Fo,m}}\right]^2\Big\}+\left[(N_{Bi}-1)\sqrt{N_{Fo,m}}-\frac{1-\bar{r}}{2\sqrt{N_{Fo,m}}}\right]\times$$

$$\times\exp\Big\{(N_{Bi}-1)^2 N_{Fo,m}+(1-\bar{r})(N_{Bi}-1)-\left[\frac{1-\bar{r}}{2\sqrt{N_{Fo,m}}}+(N_{Bi}-1)\sqrt{N_{Fo,m}}\right]^2\Big\}. \quad (20)$$

The solution simplifies greatly for $N_{Bi}=1$:

$$\bar{T}(\bar{r}, N_{Fo})\Big|_{\substack{N_{Fo}\to 0\\ N_{Bi}=1}}=\frac{1}{\bar{r}\sqrt{\pi}}\int_0^{Fo}(T_a\, N_{Fo}^*)\left\{\exp\left[-\frac{(1-\bar{r})^2}{4N_{Fo,m}}-\exp\left[-\frac{(1+\bar{r})^2}{4N_{Fo,m}}\right]\right\}\frac{dN_{Fo}^*}{\sqrt{N_{Fo,m}}}. \quad (21)$$

With the expressions (15), (16), and (19)-(21), therefore, one can calculate the temperature field in a particle during convective drying while the temperature of the gas stream varies. We note here that the problem has been solved without any constraints on the function $T_a(\tau)$ (except the condition that it as well as its derivative must be piecewise-continuous on the interval of integration). This is very important to note, because the solution obtained here can thus be extended to the more general case where the ratios of heat transfer and mass transfer coefficients as well as the latent heat of evaporation and the motive force in the drying process vary in time. Accordingly, expressions (15), (16), and (19)-(21) are also valid when the function $\bar{T}(\bar{r}, N_{Fo})$ is defined as

$$\bar{T}(\bar{r}, N_{Fo})=\frac{t(r,\tau)-t_0}{t_a(\tau)-m(\tau)\Delta P(\tau)r^*(\tau)-t_0}. \quad (22)$$

A simultaneous solution of Eqs. (16) and (19)-(21) with the equations of motion for a stream of a gaseous dispersion in a dryer chamber will produce the complete pattern of the dynamics of the temperature field in a particle along its entire trajectory so that the drying time and the moisture content in the material at the exit from the dryer can be determined. The proposed method of calculating the temperature field in a particle has been successfully used, together with the model [7, 8] of the aerodynamics of 2-phase streams in a cyclone chamber, for the design of the industrial 2-stage cyclone drying process.

NOTATION

$t(r, \tau)$, a function defining the temperature field in a particle; t_a, temperature of the ambient medium; t_0, initial temperature of a particle; R, radius of a particle; r, radius at any point in the particle; a, thermal diffusivity of a particle; α, heat-transfer coefficient; β, mass transfer coefficient; λ, thermal conductivity of the particle material; ΔP, motive force in the drying process; τ, time; θ, time at any instant on the interval $[0, \tau]$; $N_{Bi} = \alpha R/\lambda$, Biot number; $N_{Fo} = a\tau/R^2$, Fourier number; $N_{Fo}^* = a\theta/R^2$, instantaneous Fourier number; $N_{Fo}' = N_{Fo} - N_{Fo}^*$; $m = \beta/\alpha$; $\bar{T}(\bar{r}, N_{Fo}) = T(r, \tau)/T_a(\tau)$; $T_a(N_{Fo}^*) = T_a(\theta)/T_a(\tau)$.

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